Linear Algebra, Winter 2022

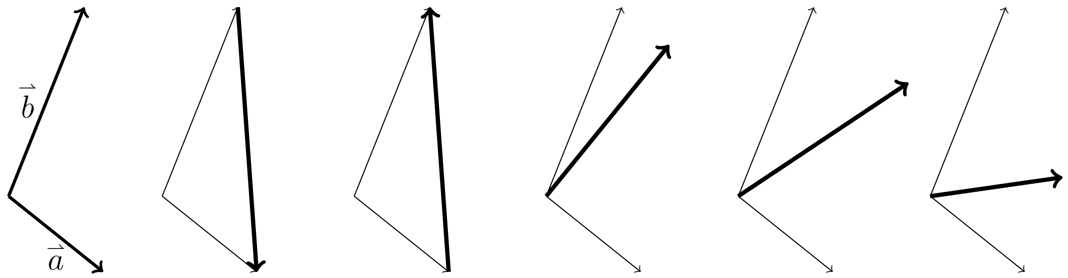
List 5

Review for Celebration of Knowledge 1

106. If $\vec{a} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ and $\vec{b} = [1, 0, -1]$ and $\vec{c} = 12\hat{k}$, calculate $\vec{a} + 4\vec{b} - \frac{1}{2}\vec{c}$. $\boxed{7\hat{\imath} + 3\hat{\jmath} - 10\hat{k}}$

107. Calculate $(9\hat{\imath}+4\hat{k})\cdot(5\hat{\imath}-\hat{\jmath}+2\hat{k}) = \boxed{53}$ and $(9\hat{\imath}+4\hat{k})\times(5\hat{\imath}-\hat{\jmath}+2\hat{k}) = \boxed{4\hat{\imath} + 2\hat{\jmath} - 9\hat{k}}$

108. Setup Picture 1 Picture 2 Picture 3 Picture 4 Picture 5



Which picture shows $\vec{a} + \vec{b}$? $\boxed{4}$ Which shows $\vec{a} - \vec{b}$? $\boxed{2}$

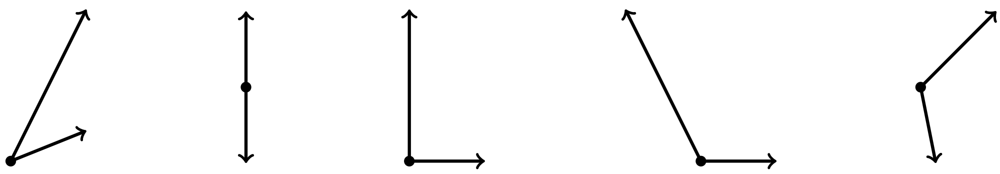
109. Find the cosine of the angle between $10\hat{\imath}+\hat{\jmath}$ and $\hat{\imath}+10\hat{\jmath}$. $\boxed{\frac{20}{101}}$ Old file said $\frac{10}{\sqrt{101}}$.

☆110. If $\vec{a}$ and $\vec{b}$ point in the same direction, $4\vec{c}$ and $8\vec{b}$ have the same length, and $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, find the angle between $\vec{a}$ and $\vec{c}$.

$$\cos(\text{angle between } \vec{a} \text{ and } \vec{c}) = \frac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|(2|\vec{b}|)} = \frac{|\vec{a}||\vec{b}|\cos(0)}{2|\vec{a}||\vec{b}|} = \frac{1}{2},$$

so the angle is $\boxed{60^\circ \text{ or } \frac{\pi}{3}}$.

111. Which picture(s) below have $\vec{u}\cdot\vec{v} = 0$? $\boxed{\text{c}}$ Which have $\vec{u}\cdot\vec{v} > 0$? $\boxed{\text{a, e}}$



(a) (b) (c) (d) (e)

☆112. If $A = (0, 0)$ and $B = (4, 3)$, find all possible positions for the point C such that ABC is a right isosceles triangle (that is, two of its sides have the same length).

113. Write $18\hat{\imath}+\hat{\jmath}$ as a linear combination of $\vec{v} = \hat{\imath}+2\hat{\jmath}$ and $\vec{w} = 2\hat{\imath}-3\hat{\jmath}$. $\boxed{[18, 1] = 8\vec{v} + 5\vec{w}}$

114. Write $\begin{bmatrix} -20 \\ 12 \\ -32 \end{bmatrix}$ as a linear combination of $\vec{a} = \begin{bmatrix} 15 \\ -9 \\ 24 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 10 \\ 12 \\ -8 \end{bmatrix}$. $\boxed{-\frac{4}{3}\vec{a} + 0\vec{b}}$

115. Write $\begin{bmatrix} 17 \\ -13 \\ 63 \end{bmatrix}$ as a linear combination of $\vec{u} = \begin{bmatrix} 9 \\ 1 \\ 25 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
$\boxed{2\vec{u} + 7\vec{v} - 22\vec{w}}$

116. Are the vectors[1] $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 10 \\ -4 \end{bmatrix}$ linear dependent or linear independent? $\boxed{\text{independent}}$

117. Are the vectors $\begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 10 \\ -4 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ linear dependent or linear independent? $\boxed{\text{dependent}}$

118. Determine whether each of the following collections of vectors are linear independent or linearly dependent:

(a) $\{[6,2]\}$ $\boxed{\text{independent}}$

(b) $\{[6,2], \quad [3,0]\}$ $\boxed{\text{independent}}$

(c) $\{[6,2], \quad [3,0], \quad [0,1]\}$ $\boxed{\text{dependent}}$

(d) $\{[6,2], \quad [3,1]\}$ $\boxed{\text{dependent}}$

(e) $\{[6,2], \quad [3,1], \quad [0,1]\}$ $\boxed{\text{dependent}}$

(f) $\{[6,2], \quad [3,1], \quad [9,3]\}$ $\boxed{\text{dependent}}$

☆119. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent, determine whether each of the following collections of vectors are linear independent or linearly dependent:

(a) $\{\vec{u}, \vec{v}\}$ $\boxed{\text{independent}}$

(b) $\{\vec{u}, \vec{v}, \vec{u}+\vec{v}\}$ $\boxed{\text{dependent}}$

(c) $\{\vec{u}, \vec{v}, \vec{u}+\vec{w}\}$ $\boxed{\text{independent}}$

(d) $\{\vec{u}, \vec{v}, 3\vec{w}\}$ $\boxed{\text{independent}}$

120. Which of the following lines is parallel to the line $\begin{cases} x = 9 + 8t \\ y = 11 - 6t \\ z = 1 + 10t \end{cases}$?

$\boxed{\text{(A)}} \begin{cases} x = 1 + 4t \\ y = -7 - 3t \\ z = 2 + 5t \end{cases}$ (B) $\begin{cases} x = 7 + 8t \\ y = 12 - 4t \\ z = 4t \end{cases}$ (C) $\begin{cases} x = 2 - 4t \\ y = 6 - 3t \\ z = 4 + 5t \end{cases}$ (D) $\begin{cases} x = 8 + 9t \\ y = -6 + 11t \\ z = 10 + t \end{cases}$

121. Which line from Task 120 is *parallel* to the plane

$$4(x-7) - 2(y-9) + 2(z+3) = 0? \qquad \boxed{\text{(C)}}$$

122. Which line from Task 120 is *perpendicular* to the plane from Task 121? $\boxed{\text{(B)}}$

[1] Technically, this should ask whether the *collection* (or *set*) of vectors $\{[5,2], [10,-4]\}$ is a linearly dependent collection or a linearly independent collection. But it is common to say that "$\vec{u}$ and $\vec{v}$ are linearly (in)dependent" when the set $\{\vec{u}, \vec{v}\}$ is linearly (in)dependent.

123. Find the intersection of the line $\begin{cases} x = 1+t \\ y = 2-2t \\ z = 8-5t \end{cases}$ and the plane $8x+2y-z=10$.

$\boxed{(\frac{5}{3}, \frac{5}{3}, \frac{2}{3})}$

124. (a) Find the intersection of the lines

$$\begin{array}{llll} L_1: & x = 1+9t, & y = 13, & z = 7+4t \\ L_2: & x = 3+5s, & y = 18-s, & z = 9+2s. \end{array}$$

$t = 3$ and $s = 5$ both give the point $\boxed{(28, 13, 19)}$.

(b) Find a vector that is perpendicular to both lines. See **Task 107**. $\boxed{[4, 2, -9]}$

(c) Give an equation for the plane that contains L_1 and L_2.

$\boxed{4(x-28)+2(y-13)-9(z-19)=0}$

125. What are the dimensions of $\begin{bmatrix} 7 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 6 & -33 & 2 \\ 0 & 0 & 0 \end{bmatrix}$? $\boxed{1 \times 3}$

126. Calculate the product in Task 125. $\boxed{\begin{bmatrix} 14 & -77 & \frac{14}{3} \end{bmatrix}}$

127. If $A = \begin{bmatrix} 4 & 0 & 0 & -2 & -6 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 0 & 19 & -8 \end{bmatrix} B$, and matrix A is invertible, what are the dimensions of matrix A and the dimension of matrix B? A has dimensions $\boxed{3 \times 3}$, and B has dimensions $\boxed{5 \times 3}$.

128. Multiply the following matrices, or state that the product does not exist.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} = \boxed{\begin{bmatrix} 21 & 24 & 27 \\ 47 & 54 & 61 \end{bmatrix}}$

(b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \\ 9 & 10 \end{bmatrix}$ $\boxed{\text{doesn't exist}}$

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 0 \\ 0 & 0 \end{bmatrix}}$

(d) $\begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 5 \\ 1 & 1 \end{bmatrix}}$

(e) $\begin{bmatrix} 1 & 0 & 2 \\ 5 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 5 & 0 & 5 \end{bmatrix}$ $\boxed{\text{does't exist}}$

(f) $\begin{bmatrix} 1 & 0 & 2 \\ 5 & 0 & 5 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 1 & 3 \\ 25 & 5 & 15 \end{bmatrix}}$

(g) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 15 & 8 & -2 \\ 3 & 5 & 1 \\ 9 & 9 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 30 & 16 & -4 \\ 6 & 10 & 2 \\ 18 & 18 & 4 \end{bmatrix}}$

129. Which of the following are linear transformations?

(a) $f(x, y) = (x + 10, y)$ $\boxed{\text{no}}$

(b) $f(x, y) = (10x, y)$ $\boxed{\text{yes}}$

(c) $f(x, y) = (x + 2y, x - 2y)$ $\boxed{\text{yes}}$

(d) $f(x, y) = (x + 2y, y - 2x)$ $\boxed{\text{yes}}$

(e) $f(x, y) = (100x^2, y)$ $\boxed{\text{no}}$

130. If $f(x, y) = (x + y, 0)$ and $g(x, y) = (5x - y, x + y)$, give a formula for $f(g(x, y))$ and a formula for $g(f(x, y))$. This can be done using 128(c) and 128(d), but it can also be done just by standard function rules. $\boxed{f(g(x, y)) = (6x, 0)}$ and $\boxed{g(f(x, y)) = (5x + 5y, x + y)}$

131. Calculate the determinant and the inverse of $\begin{bmatrix} 5 & 1 \\ 8 & 2 \end{bmatrix}$.

$\boxed{\det(M) = 2}$ and $\boxed{M^{-1} = \begin{bmatrix} 1 & -1/2 \\ 4 & 5/2 \end{bmatrix}}$

132. Calculate the determinant of $\begin{bmatrix} 11 & 10 & 7 \\ 1 & 0 & 0 \\ 11 & 18 & 15 \end{bmatrix}$.

$$11 \cdot \det \begin{pmatrix} 0 & 0 \\ 18 & 15 \end{pmatrix} - 10 \cdot \det \begin{pmatrix} 1 & 0 \\ 11 & 15 \end{pmatrix} + 7 \cdot \det \begin{pmatrix} 1 & 0 \\ 11 & 18 \end{pmatrix} = -10(15) + 7(18) = \boxed{-24}$$

133. If A is a 6×6 matrix with $\det(A) = 5$, and B is a 6×2 matrix, which of the following exist?

(a) $2A + B$ $\boxed{\text{does not exist}}$

(b) $3B + A$ $\boxed{\text{does not exist}}$

(c) AB $\boxed{\text{exists}}$

(d) BA $\boxed{\text{does not exist}}$

(e) $I_{6\times6} + A$ $\boxed{\text{exists}}$

(f) $I_{6\times6} + B$ $\boxed{\text{does not exist}}$

(g) $I_{6\times6}A$ $\boxed{\text{exists}}$

(h) $I_{6\times6}B$ $\boxed{\text{exists}}$

(i) A^{-1} $\boxed{\text{exists}}$

(j) B^{-1} $\boxed{\text{does not exist}}$

(k) $A^{-1} + B^{-1}$ $\boxed{\text{does not exist}}$

(ℓ) $A^{-1}B$ $\boxed{\text{exists}}$

134. Solve the following systems of equations, if they have solutions.

(a) $\begin{cases} x + 8y = 9 \\ x - 12y = -1 \end{cases}$ $\boxed{x = 5,\ y = -1}$

(b) $\begin{cases} 10x - 4y = 5 \\ 5x - 2y = 10 \end{cases}$ $\boxed{\text{no solution}}$

☆(c) $\begin{cases} 10x - 4y = 10 \\ 5x - 2y = 5 \end{cases}$ Any (x, y) with $y = \frac{5x-5}{2}$ is a solution.

135. Calculate the rank of $\begin{bmatrix} 6 & 2 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}$...

Using **Task 118a-c**, we have that the rank is $\boxed{2}$ because $\{[6, 2], [3, 0]\}$ is linearly independent but the collection of all three rows is dependent.

... and the rank of $\begin{bmatrix} 6 & 2 \\ 3 & 1 \\ 9 & 3 \end{bmatrix}$.

Using **Task 118d-e**, this is rank $\boxed{1}$.

136. Calculate the rank of $\begin{bmatrix} 6 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ and the rank of $\begin{bmatrix} 6 & 3 & 9 \\ 2 & 1 & 3 \end{bmatrix}$.

This is the same as the previous task. $\boxed{2}$ and $\boxed{1}$

137. The determinant of $\begin{bmatrix} -4 & 19 & -10 & 6 \\ -10 & 19 & 19 & -5 \\ 10 & 10 & 8 & -5 \\ 2 & 7 & -12 & 5 \end{bmatrix}$ is 36. What is its rank? $\boxed{4}$